基于边状态枚举计算多状态网络可靠度动态界

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摘 要 为降低计算多状态网络可靠度的复杂性,综合考虑网络中具有多态性的边处于各中间状态的概率及从某中间状态转换到相邻状态对网络性能的影响,提出了一种基于边状态枚举计算多状态网络可靠度上下界的算法。该算法首先令网络中各边仅取完全工作和完全失效两种状态,将处于中间状态的概率分别叠加到完全工作和完全失效状态的概率上,得到可靠度上下界的初始值;而后按照对可靠度影响递减的顺序迭代枚举边的中间状态,通过集合间的比较,计算可靠度上下界的改变值,同时获得不断减小的可靠度上界和不断增加的可靠度下界,使其最终收敛于可靠度精确值。该算法不需提前求取网络d-最小割(路)集,且枚举较少的网络状态即可得到紧凑的可靠度上下界。相关引理的证明及算例分析验证了该算法的正确性和有效性。

关键词 多状态网络,随机流量网络,可靠度界,边状态枚举

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Dynamic Bounding Algorithm for Approximating Multi-state Network Reliability Based on Arc State Enumeration

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Abstract In order to reduce complexity of calculating multi-state network reliability, considering stay probability and effect on network capability when transfering to adjacent state of multimode arc in each intermediate state, an algorithm for calculating multi-state network reliability dynamic bounds based on arc state enumeration was proposed. Firstly, assuming arcs can be in either of two states: operating or failed, obtained initial reliability upper and lower bounds respectively by adding probability of each arc in all intermediate states to probability of operating or failed state. Then, iteratively enumerated intermediate states in order of decreasing effect to network reliability, and calculated variation value of upper and lower bounds by comparison operation of sets, while it derived a series of decreasing upper bounds and a series of increasing lower bounds simultaneously, guaranteed to enclose the exact reliability value. The algorithm does not require a priori the d-minimal cuts or d-minimal puts of the multi-state network, and can ensure an exact difference between the upper and lower bounds with enumerating fewer network states. Related lemma warrant and example analysis verify correctness and effectiveness of the algorithm.

Keywords Multi-state network, Stochastic-flow network, Reliability bound, Arc state enumeration

1 引言

随着社会和科学技术的不断发展,网络化已成为一种趋势,网络可靠性研究在分布式系统中的作用与日俱增,其计算方法已成为国内外研究的热点。近年来,除两状态网络模型外,国内外学者对多状态网络(又称随机流量网络)可靠性的研究也越来越多。多状态网络中,元素的工作性能是逐渐衰退的,从工作到失效经历了许多中间状态,因此计算其可靠度更加繁琐[1-4]。可通过状态树搜索[5]、空间分解[6,7]、多元决策图[8]及 d-最小割(路)集[9,10]等计算可靠度精确值,但其均属于非多项式(NP)算法问题。由于工程实际中允许计算结果有一定的误差,因此探索高效的近似算法就成为一个研究热点。归纳起来,近似算法主要分为两类:基于随机模拟的Monte-Carlo估计算法[11-13]和可靠度界的确定方法[14-29]。其中可靠度界相关算法又包括基于d-最小割(路)集的算法[14-21]和不需d-最小割(路)集的算法[22-29]。前者的相关研究主要有:文献[14-19]通过选取部分d-最小割(路)集计算可靠度下界;文献[20,21]分别扩展两状态网络的ESP界、EDP界和LQ界,得到多状态网络的MESP界、MEDP界和MLQ界。这些可靠度界的主要特点有:(1)确定的可靠度界为单一可靠度界,无法同时得到上下界;(2)得到的可靠度界为静态界,即产生的界是确定的,不随算法运算次数的改变而趋近于

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真实值；(3)算法需提前求取 d-最小割(路)集，而 d-最小割(路)集的求取又需以最小割(路)集为基础。无论求最小割(路)集，还是在此基础上运用组合数学理论求 d-最小割(路)集，均为 NP 算法问题。因此，这类可靠度界的实际应用较为困难。不需 d-最小割(路)集的可靠度界确定方法主要有两类：空间分解[22]及空间截尾[23-29]。前者在空间分解精确算法[6]的基础上，通过事先设定极小值 α 来控制分解过程，同时获得可靠度上下界，算法中 α 的数值直接决定了分解的深入程度，但其数值的大小又与可靠度上下界无明确关系，这使得算法在实际应用中无法根据所需可靠度的精确度来确定 α 数值。因此，已有多状态网络可靠度界的相关算法中，空间截尾法最具有实际应用价值，并引起了广泛研究。通过扩展两状态网络可靠度近似计算中的空间截尾算法，针对多状态网络出现了许多具体算法，常用的有 ORDER-M[23]、YANG-KUBAT[24]、POSE[25]、ORDER-M2/POSE2[26,27] 和 ORDEP-M-π[28,29]。这些算法的不同之处在于如何依概率递减的次序产生最可能的网络状态，使其概率和达到状态空间一定的覆盖率；其共同之处在于均有一个重要前提：假设网络的极少数个状态发生的概率之和占据大部分状态空间。因此，算法要获得误差小于 ε_0 的上下界需枚举的网络状态发生的概率之和应不小于 $1-\varepsilon_0$。若各网络状态概率相差不大，则需枚举较多网络状态。如网络中各多态边处于各状态的概率相等，则需枚举的网络状态数量占总数量的 $1-\varepsilon_0$，对于较小的 ε_0 其数目仍较多。多状态网络中的边除完全工作和完全失效状态外，还有可能处于性能衰退的中间状态，这些中间状态对网络可靠度的影响取决于边处于该状态的概率及边从该状态转换到相邻状态时对网络性能的影响。综合考虑这两种因素，本文提出了通过边状态枚举求取多状态网络可靠度上下界的算法。

2 多状态网络描述

$G(V,E)$ 表示节点集合为 $V=\{v_1(s),v_2,\cdots,v_i,\cdots,v_{|V|}(t)\}$($s,t$ 分别表示源点和汇点)，边集合为 $E=\{e_1,e_2,\cdots,e_i,\cdots,e_{|E|}\}$ 的网络，$|\cdot|$ 表示集合“$\cdot$”中元素个数；$B_i=\{b_{i,1},b_{i,2},\cdots,b_{i,j_i},\cdots,b_{i,n_i}\}$($0=b_{i,1}<b_{i,2}<\cdots<b_{i,j_i}<\cdots<b_{i,n_i}$)表示边 e_i 的容量(允许通过的最大流量)取值集合，边 e_i 具有 n_i 种状态，其容量是一随机变量，以已知概率在 B_i 中取值，其中：$b_{i,1}=0$ 和 b_{i,n_i} 分别表示边 e_i 处于完全失效和完全工作状态，其余 b_{i,j_i}($1<j_i<n_i$)表示边 e_i 处于性能衰退的中间状态，对应取值概率集合 $P_i=\{p_{i,1},p_{i,2},\cdots,p_{i,j_i},\cdots,p_{i,n_i}\}$，其中 p_{i,j_i} 表示边 e_i 在集合 B_i 中取容量值 b_{i,j_i} 的概率，$\sum_{j_i=1}^{n_i}p_{i,j_i}=1$；$\chi=B_1\times B_2\times\cdots\times B_i\times\cdots\times B_{|E|}$ 表示 $G(V,E)$ 的状态空间，$\times$ 为笛卡尔积；$\boldsymbol{X}=(x_1,x_2,\cdots,x_i,\cdots,x_{|E|})\in\chi$ 表示网络状态的向量，其中：$x_i\in B_i$ 表示边 e_i 的当前容量，则网络处于状态 $\boldsymbol{X}$ 的概率 $\Pr(\boldsymbol{X})=\prod_{i=1,x_i=b_{i,j_i}}^{|E|}p_{i,j_i}$；$d$ 表示网络的需求流量，即要求汇点 t 能够接收到源点 s 所发出信息的最低流量；$f(\boldsymbol{X})$ 表示网络状态为 $\boldsymbol{X}$ 时，源点 s 和汇点 t 之间允许通过的最大流量，如 $f(\boldsymbol{X})\geqslant d$，称 $\boldsymbol{X}$ 为有效状态，如 $f(\boldsymbol{X})<d$，称 $\boldsymbol{X}$ 为无效状态；多状态网络 $G(V,E)$ 在需求流量为 d 时，其可靠度 R 定义为能够将 d 水平需求流量从源点 s 通过具有多态性的边传输到汇点 t 的概率；对于状态 $\boldsymbol{X}=(x_1,x_2,\cdots,x_i,\cdots,x_{|E|})$，$\boldsymbol{Y}=(y_1,y_2,\cdots,y_i,\cdots,y_{|E|})$，定义：(1)$\boldsymbol{X}\leqslant(\geqslant)\boldsymbol{Y}$，当且仅当 $x_i\leqslant(\geqslant)y_i$ 对于所有的 $i(i=1,2\cdots,|E|)$ 成立；(2)$\boldsymbol{X}<(>)\boldsymbol{Y}$，当且仅当 $x_i\leqslant(\geqslant)y_i(i=1,2\cdots,|E|)$，且其中至少存在一个 i 相应式中等号不成立。

3 算法原理

为便于描述，记 $\boldsymbol{X}_{e_i}^{x_i}$ 表示将网络状态 $\boldsymbol{X}=(x_1,x_2,\cdots,x_i,\cdots,x_{|E|})$ 中边 e_i 的容量取值 x_i 去掉后对应的 $|E|-1$ 维向量，即 $\boldsymbol{X}_{e_i}^{x_i}=(x_1,x_2,\cdots,x_{i-1},x_{i+1},\cdots,x_{|E|})$，记 $\boldsymbol{X}_{e_i}^{x_i}=\boldsymbol{X}/(e_i,x_i)$，$\boldsymbol{X}=(e_i,x_i)\cdot\boldsymbol{X}_{e_i}^{x_i}$，$\Pr(\boldsymbol{X}_{e_i}^{x_i})=\prod_{k=1,k\neq i,x_k=b_{k,j_k}}^{|E|}p_{k,j_k}$，$\Pr(\boldsymbol{X})=\prod_{k=1,x_k=b_{k,j_k}}^{|E|}p_{k,j_k}$，则 $\Pr(\boldsymbol{X})=p_{i,j_i}|_{x_i=b_{i,j_i}}\prod_{k=1,k\neq i,x_k=b_{k,j_k}}^{|E|}p_{k,j_k}=p_{i,j_i}|_{x_i=b_{i,j_i}}\Pr(\boldsymbol{X}_{e_i}^{x_i})$；对于网络状态组成的集合 Ω，取其中所有边 e_i 的容量取值 x_i 为 b_{i,j_i} 的状态组成其子集合，去掉该子集合中所有元素的边 e_i 的容量取值得到的所有 $|E|-1$ 维向量组成的集合记为 $\Omega_{e_i}^{b_{i,j_i}}$，即 $\Omega_{e_i}^{b_{i,j_i}}=\{\boldsymbol{X}_{e_i}^{b_{i,j_i}}\mid\boldsymbol{X}_{e_i}^{b_{i,j_i}}=\boldsymbol{X}/(e_i,b_{i,j_i}),\boldsymbol{X}\in\Omega,x_i=b_{i,j_i}\}$，记 $\Omega_{e_i}^{b_{i,j_i}}=\Omega/(e_i,b_{i,j_i})$，$\{\boldsymbol{X}|\boldsymbol{X}\in\Omega,x_i=b_{i,j_i}\}=(e_i,b_{i,j_i})\cdot\Omega_{e_i}^{b_{i,j_i}}$，$\Pr(\Omega_{e_i}^{b_{i,j_i}})=\sum_{X_{e_i}^{b_{i,j_i}}\in\Omega_{e_i}^{b_{i,j_i}}}\Pr(\boldsymbol{X}_{e_i}^{b_{i,j_i}})$，$\Pr(\Omega)=\sum_{X\in\Omega}\Pr(\boldsymbol{X})$，则 $\Omega=\bigcup_{j_i=1}^{n_i}\{\boldsymbol{X}|\boldsymbol{X}\in\Omega,x_i=b_{i,j_i}\}=\bigcup_{j_i=1}^{n_i}((e_i,b_{i,j_i})\cdot\Omega_{e_i}^{b_{i,j_i}})$，$\Pr(\{\boldsymbol{X}|\boldsymbol{X}\in\Omega,x_i=b_{i,j_i}\})=p_{i,j_i}\Pr(\Omega_{e_i}^{b_{i,j_i}})$，$\Pr(\Omega)=\sum_{j_i=1}^{n_i}\Pr(\{\boldsymbol{X}|\boldsymbol{X}\in\Omega,x_i=b_{i,j_i}\})=\sum_{j_i=1}^{n_i}(p_{i,j_i}\Pr(\Omega_{e_i}^{b_{i,j_i}}))$。

3.1 基于边的部分中间状态计算可靠度上下界

由多状态网络 $G(V,E)$ 在需求流量为 d 时的可靠度 R 的定义可知：

$$\begin{aligned}R&=\Pr(\{\boldsymbol{X}|\boldsymbol{X}\in\chi,f(\boldsymbol{X})\geqslant d\})=\Pr(\Omega)\\&=\sum_{j_i=1}^{n_i}(p_{i,j_i}\Pr(\Omega_{e_i}^{b_{i,j_i}}))\end{aligned}\tag{1}$$

式中，$\Omega=\{\boldsymbol{X}|\boldsymbol{X}\in\chi,f(\boldsymbol{X})\geqslant d\}$ 称为网络的有效状态空间，即网络状态空间中所有的有效状态组成的集合。

忽略边 e_i 的部分中间状态，将边 e_i 的容量取值集合取为 $\bar{B}_i=\{\bar{b}_{i,1},\bar{b}_{i,2},\cdots,\bar{b}_{i,\bar{j}_i},\cdots,\bar{b}_{i,\bar{n}_i}\}$($b_{i,1}=\bar{b}_{i,1}<\bar{b}_{i,2}<\cdots<\bar{b}_{i,\bar{j}_i}\cdots<\bar{b}_{i,\bar{n}_i}=b_{i,n_i}$)，其中 $\bar{B}_i\subseteq B_i$，即边 e_i 的容量取值集合 $\bar{B}_i$ 变为 B_i 的子集，对应的状态数量 $\bar{n}_i\leqslant n_i$，对应的网络状态空间 $\bar{\chi}=B_1\times B_2\times\cdots\times\bar{B}_i\times\cdots\times B_{|E|}\subseteq\chi$；取值概率集合 $\bar{P}_i=\{\bar{p}_{i,1},\bar{p}_{i,2},\cdots,\bar{p}_{i,\bar{j}_i},\cdots,\bar{p}_{i,\bar{n}_i}\}$；网络有效状态集合 $\bar{\Omega}=\{\boldsymbol{X}\mid\boldsymbol{X}\in\bar{\chi},f(\boldsymbol{X})\geqslant d\}\subseteq\Omega$。由 $\bar{\Omega}$ 得到的可靠度记为 $\bar{R}$，则基于式(1)，得：

$$\begin{aligned}\bar{R}&=\Pr(\{\boldsymbol{X}|\boldsymbol{X}\in\bar{\chi},f(\boldsymbol{X})\geqslant d\})=\Pr(\bar{\Omega})\\&=\sum_{\bar{j}_i=1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\bar{\Omega}_{e_i}^{\bar{b}_{i,\bar{j}_i}}))\end{aligned}\tag{2}$$

分别取 $\bar{p}_{i,\bar{j}_i}=\begin{cases}p_{i,1}, & \bar{j}_i=1\\ \sum_{\bar{b}_{i,\bar{j}_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{j}_i}}p_{i,j_i}, & 1<\bar{j}_i\leqslant\bar{n}_i\end{cases}$，$\bar{p}_{i,\bar{j}_i}=\begin{cases}\sum_{\bar{b}_{i,\bar{j}_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{j}_i+1}}p_{i,j_i}, & 1\leqslant\bar{j}_i<\bar{n}_i\\ p_{i,n_i}, & \bar{j}_i=\bar{n}_i\end{cases}$ 代入式(2)，得到的 $\bar{R}$ 的值分别记为 $\hat{R}$ 和 $\check{R}$。下面证明 $\hat{R}\geqslant R\geqslant\check{R}$。

引理 1　基于上述定义，有$\hat{R}\geqslant R\geqslant\check{R}$。

证明：因 $\bar{\Omega}=\{\boldsymbol{X}|\boldsymbol{X}\in\bar{\chi},f(\boldsymbol{X})\geqslant d\}=\{\boldsymbol{X}|\boldsymbol{X}\in\chi,x_i\in\bar{B}_i,f(\boldsymbol{X})\geqslant d\}=\{\boldsymbol{X}|\boldsymbol{X}\in\Omega,x_i\in\bar{B}_i\}$，则 $\bar{\Omega}_{e_i}^{\bar{b}_{i,\bar{j}_i}}=\{\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}|\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}=\boldsymbol{X}/(e_i,\bar{b}_{i,\bar{j}_i}),\boldsymbol{X}\in\bar{\Omega},x_i=\bar{b}_{i,\bar{j}_i}\}=\{\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}|\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}=\boldsymbol{X}/(e_i,\bar{b}_{i,\bar{j}_i}),\boldsymbol{X}\in\Omega,x_i\in\bar{B}_i,x_i=\bar{b}_{i,\bar{j}_i}\}$。如 $x_i=\bar{b}_{i,\bar{j}_i}$，则 $x_i\in\bar{B}_i$，因此 $\bar{\Omega}_{e_i}^{\bar{b}_{i,\bar{j}_i}}=\{\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}|\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}=\boldsymbol{X}/(e_i,\bar{b}_{i,\bar{j}_i}),\boldsymbol{X}\in\Omega,x_i=\bar{b}_{i,\bar{j}_i}\}=\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}$，即 $\bar{\Omega}_{e_i}^{\bar{b}_{i,\bar{j}_i}}=\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}$，代入式(2)，有

$$\bar{R}=\sum_{\bar{j}_i=1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}})) \tag{3}$$

将 $\bar{p}_{i,\bar{j}_i}=\begin{cases}p_{i,1}, & \bar{j}_i=1\\ \sum\limits_{\bar{b}_{i,\bar{j}_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{j}_i}}p_{i,j_i}, & 1<\bar{j}_i\leqslant\bar{n}_i\end{cases}$ 代入式(3)，得

$$\hat{R}=\sum_{\bar{j}_i=1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))=p_{i,1}\Pr(\Omega_{e_i}^{b_{i,1}})+\sum_{\bar{j}_i=2}^{\bar{n}_i}((\sum_{\bar{b}_{i,\bar{j}_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{j}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}})) \tag{4}$$

对于 $\bar{b}_{i,\bar{j}_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{j}_i}$，任一 $\boldsymbol{X}_{e_i}^{b_{i,j_i}}\in\Omega_{e_i}^{b_{i,j_i}}$，有 $(e_i,\bar{b}_{i,\bar{j}_i})\cdot\boldsymbol{X}_{e_i}^{b_{i,j_i}}\geqslant(e_i,b_{i,j_i})\cdot\boldsymbol{X}_{e_i}^{b_{i,j_i}}$，则 $f((e_i,\bar{b}_{i,\bar{j}_i})\cdot\boldsymbol{X}_{e_i}^{b_{i,j_i}})\geqslant f((e_i,b_{i,j_i})\cdot\boldsymbol{X}_{e_i}^{b_{i,j_i}})\geqslant d$，即 $\boldsymbol{X}_{e_i}^{b_{i,j_i}}\in\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}$，有 $\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}\supseteq\Omega_{e_i}^{b_{i,j_i}}$，$\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}})\geqslant\Pr(\Omega_{e_i}^{b_{i,j_i}})$，代入式(4)，则

$$\begin{aligned}\hat{R}&=p_{i,1}\Pr(\Omega_{e_i}^{b_{i,1}})+\sum_{\bar{j}_i=2}^{\bar{n}_i}((\sum_{\bar{b}_{i,\bar{j}_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{j}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))\\&\geqslant p_{i,1}\Pr(\Omega_{e_i}^{b_{i,1}})+\sum_{\bar{j}_i=2}^{\bar{n}_i}((\sum_{\bar{b}_{i,\bar{j}_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{j}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,j_i}}))\\&=\sum_{j_i=1}^{n_i}(p_{i,j_i}\Pr(\Omega_{e_i}^{b_{i,j_i}}))=R\end{aligned} \tag{5}$$

即$\hat{R}\geqslant R$。

将 $\bar{p}_{i,\bar{j}_i}=\begin{cases}\sum\limits_{\bar{b}_{i,\bar{j}_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{j}_i+1}}p_{i,j_i}, & 1\leqslant\bar{j}_i<\bar{n}_i\\ p_{i,n_i}, & \bar{j}_i=\bar{n}_i\end{cases}$ 代入式(3)，得

$$\check{R}=\sum_{\bar{j}_i=1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))=\sum_{\bar{j}_i=1}^{\bar{n}_i-1}((\sum_{\bar{b}_{i,\bar{j}_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{j}_i+1}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+p_{i,n_i}\Pr(\Omega_{e_i}^{b_{i,n_i}}) \tag{6}$$

对于 $\bar{b}_{i,\bar{j}_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{j}_i+1}$，任一 $\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}\in\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}$，有 $(e_i,b_{i,j_i})\cdot\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}\geqslant(e_i,\bar{b}_{i,\bar{j}_i})\cdot\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}$，则 $f((e_i,b_{i,j_i})\cdot\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}})\geqslant f((e_i,\bar{b}_{i,\bar{j}_i})\cdot\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}})\geqslant d$，即 $\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{j}_i}}\in\Omega_{e_i}^{b_{i,j_i}}$，有 $\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}\subseteq\Omega_{e_i}^{b_{i,j_i}}$，$\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}})\leqslant\Pr(\Omega_{e_i}^{b_{i,j_i}})$，代入式(6)，则

$$\begin{aligned}\check{R}&=\sum_{\bar{j}_i=1}^{\bar{n}_i-1}((\sum_{\bar{b}_{i,\bar{j}_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{j}_i+1}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+p_{i,n_i}\Pr(\Omega_{e_i}^{b_{i,n_i}})\\&\leqslant\sum_{\bar{j}_i=1}^{\bar{n}_i-1}((\sum_{\bar{b}_{i,\bar{j}_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{j}_i+1}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,j_i}}))+p_{i,n_i}\Pr(\Omega_{e_i}^{b_{i,n_i}})\\&=\sum_{j_i=1}^{n_i}(p_{i,j_i}\Pr(\Omega_{e_i}^{b_{i,j_i}}))=R\end{aligned} \tag{7}$$

即$\check{R}\leqslant R$。

综上所述，有$\hat{R}\geqslant R\geqslant\check{R}$得证。

引理 1 表明，忽略边 e_i 的某些中间状态，取其容量取值集合的子集，枚举相对应的所有网络有效状态，将处于忽略的中间状态的概率叠加到相邻的未忽略的状态概率上，将网络有效状态概率相加，即可得到可靠度上下界。极端情况下，边 e_i 仅取完全失效状态和完全工作状态 b_{i,n_i}，此时只需枚举 $2^{|E|}$ 个网络状态即可得出所有对应的网络有效状态，将边 e_i 处于各中间状态 $b_{i,j_i}(1<j_i<n_i)$ 的概率分别叠加到处于 b_{i,n_i} 和 $b_{i,1}$ 的概率上，即可分别得到可靠度上下界。

3.2 迭代增加枚举边的中间状态计算可靠度动态上下界

假设边 e_i 的某中间状态 $b_{i,k_i}\in B_i$ 且 $b_{i,k_i}\notin\bar{B}_i$，$\bar{b}_{i,\bar{k}_i-1}<b_{i,k_i}<\bar{b}_{i,\bar{k}_i}$，增加枚举该中间状态，即将 b_{i,k_i} 并入 $\bar{B}_i(\bar{B}_i'=\bar{B}_i\cup\{b_{i,k_i}\})$，基于式(2)得到的上下界，分别记为$\hat{R}'$和$\check{R}'$。下面证明$\hat{R}'$和$\check{R}'$是比$\hat{R}$和$\check{R}$更为紧凑的上下界，即$\hat{R}\geqslant\hat{R}'\geqslant R\geqslant\check{R}'\geqslant\check{R}$。

引理 2　基于上述定义，有$\hat{R}\geqslant\hat{R}'\geqslant R\geqslant\check{R}'\geqslant\check{R}$。

证明：增加 b_{i,k_i} 后，对应的 $\bar{B}_i'=\{\bar{b}_{i,1},\bar{b}_{i,2},\cdots,\bar{b}_{i,\bar{k}_i-1},b_{i,k_i},\bar{b}_{i,\bar{k}_i},\cdots,\bar{b}_{i,\bar{n}_i}\}$，将 $\bar{p}'_{i,\bar{j}_i}=\begin{cases}\bar{p}_{i,\bar{j}_i}, & \bar{j}_i\neq k_i,\bar{j}_i\neq\bar{k}_i\\ \sum\limits_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i}, & \bar{j}_i=k_i\\ \sum\limits_{b_{i,k_i}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{k}_i}}p_{i,j_i}, & \bar{j}=\bar{k}\end{cases}$

代入式(3)，得

$$\begin{aligned}\hat{R}'&=\sum_{\bar{j}_i=1}^{\bar{k}_i-1}(\bar{p}'_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{b_{i,\bar{j}_i}}))+\bar{p}'_{i,k_i}\Pr(\Omega_{e_i}^{b_{i,k_i}})+\bar{p}'_{i,\bar{k}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})+\sum_{\bar{j}_i=\bar{k}_i+1}^{n_i-1}(\bar{p}'_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))\\&=\sum_{\bar{j}_i=1}^{\bar{k}_i-1}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{b_{i,\bar{j}_i}}))+(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})+(\sum_{b_{i,k_i}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})+\sum_{\bar{j}_i=\bar{k}_i+1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}})\end{aligned} \tag{8}$$

任一 $\boldsymbol{X}_{e_i}^{b_{i,k_i}}\in\Omega_{e_i}^{b_{i,k_i}}$，因为 $\bar{b}_{i,\bar{k}_i}>b_{i,k_i}$，有 $(e_i,\bar{b}_{i,\bar{k}_i})\cdot\boldsymbol{X}_{e_i}^{b_{i,k_i}}\geqslant(e_i,b_{i,k_i})\cdot\boldsymbol{X}_{e_i}^{b_{i,k_i}}$，则 $f((e_i,\bar{b}_{i,\bar{k}_i})\cdot\boldsymbol{X}_{e_i}^{b_{i,k_i}})\geqslant f((e_i,b_{i,k_i})\cdot\boldsymbol{X}_{e_i}^{b_{i,k_i}})\geqslant d$，即 $\boldsymbol{X}_{e_i}^{b_{i,k_i}}\in\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}}$，有 $\Omega_{e_i}^{b_{i,k_i}}\subseteq\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}}$，$\Pr(\Omega_{e_i}^{b_{i,k_i}})\leqslant\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})$，代入式(8)，则

$$\begin{aligned}\hat{R}'&\leqslant\sum_{\bar{j}_i=1}^{\bar{k}_i-1}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})+(\sum_{b_{i,k_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})+\sum_{\bar{j}_i=\bar{k}_i+1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))\\&=\sum_{\bar{j}_i=1}^{\bar{k}_i-1}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})+(\sum_{\bar{j}_i=\bar{k}_i+1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}})))\\&=\sum_{\bar{j}_i=1}^{\bar{k}_i-1}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+\bar{p}_{i,\bar{k}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})+\sum_{\bar{j}_i=\bar{k}_i+1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))=\hat{R}\end{aligned} \tag{9}$$

即$\hat{R}\geqslant\hat{R}'$；因 $\bar{B}_i'$ 仍为 B_i 的子集，由引理 1 知$\hat{R}'\geqslant R$；即得$\hat{R}\geqslant\hat{R}'\geqslant R$。

将 $\bar{p}'_{i,\bar{j}_i}=\begin{cases}\bar{p}_{i,\bar{j}_i}, & \bar{j}_i\neq k_i,\bar{j}_i\neq\bar{k}_i-1\\ \sum\limits_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i}, & \bar{j}_i=k_i\\ \sum\limits_{\bar{b}_{i,\bar{k}_i-1}\leqslant b_{i,j_i}<b_{i,k_i}}p_{i,j_i}, & \bar{j}=\bar{k}_i-1\end{cases}$ 代入式(3)得

$$\check{R}'=\sum_{\bar{j}_i=1}^{\bar{k}_i-2}(\bar{p}'_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+\bar{p}'_{i,\bar{k}_i-1}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+\bar{p}'_{i,k_i}\Pr(\Omega_{e_i}^{b_{i,k_i}})+\sum_{\bar{j}_i=\bar{k}_i}^{\bar{n}_i}(\bar{p}'_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))$$

$$=\sum_{\bar{j}_i=1}^{\bar{k}_i-2}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+(\sum_{\bar{b}_{i,\bar{k}_i-1}\leqslant b_{i,j_i}<b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})+\sum_{\bar{j}_i=\bar{k}_i}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}})) \tag{10}$$

任一$\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{k}_i-1}}\in\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}}$，因为$b_{i,k_i}>\bar{b}_{i,\bar{k}_i-1}$，有$(e_i,b_{i,k_i})\cdot\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{k}_i-1}}\geqslant(e_i,\bar{b}_{i,\bar{k}_i-1})\cdot\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{k}_i-1}}$，则$f((e_i,b_{i,k_i})\cdot\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})\geqslant f((e_i,\bar{b}_{i,\bar{k}_i-1})\cdot\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})\geqslant d$，即$\boldsymbol{X}_{e_i}^{\bar{b}_{i,\bar{k}_i-1}}\in\Omega_{e_i}^{b_{i,k_i}}$，有$\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}}\subseteq\Omega_{e_i}^{b_{i,k_i}}$，$\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})\leqslant\Pr(\Omega_{e_i}^{b_{i,k_i}})$，代入式(10)，则

$$\check{R}'\geqslant\sum_{\bar{j}_i=1}^{\bar{k}_i-2}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+(\sum_{\bar{b}_{i,\bar{k}_i-1}\leqslant b_{i,j_i}<b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+\sum_{\bar{j}_i=\bar{k}_i}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))$$

$$=\sum_{\bar{j}_i=1}^{\bar{k}_i-2}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+(\sum_{\bar{b}_{i,\bar{k}_i-1}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+\sum_{\bar{j}_i=\bar{k}_i}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))$$

$$=\sum_{\bar{j}_i=1}^{\bar{k}_i-2}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+\bar{p}_{i,\bar{k}_i-1}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+\sum_{\bar{j}_i=\bar{k}_i}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))=\check{R} \tag{11}$$

即$\check{R}'\geqslant\check{R}$；因为$\bar{B}_i{}'$仍为$B_i$的子集，由引理1知$R\geqslant\check{R}'$，即得$R\geqslant\check{R}'\geqslant\check{R}$。

综上所述，$\hat{R}\geqslant\hat{R}'\geqslant R\geqslant\check{R}'\geqslant\check{R}$得证。

引理2表明，枚举边e_i的中间状态越多，得到的可靠度界越紧凑。如每枚举边e_i一个中间状态就重新枚举出网络所有的有效状态，再基于式(2)计算可靠度上下界势必会造成重复计算，因此其计算效率较低。下面通过引理3提出一种增加枚举边e_i的中间状态b_{i,k_i}后计算可靠度上下界的变化值的快速方法。

引理3 对于任意集合A和B，规定$A-B=\{\boldsymbol{X}|\boldsymbol{X}\in A,\boldsymbol{X}\notin B\}$，则$\hat{R}-\hat{R}'=(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}}-\Omega_{e_i}^{b_{i,k_i}})$，$\check{R}'-\check{R}=(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}}-\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})$。

证明：由式(4)减式(8)，得

$$\hat{R}-\hat{R}'=\sum_{\bar{j}_i=1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))-(\sum_{\bar{j}_i=1}^{\bar{k}_i-1}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})+(\sum_{b_{i,k_i}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})+\sum_{\bar{j}_i=\bar{k}_i+1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}})))$$

$$=\bar{p}_{i,\bar{k}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})-(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})-(\sum_{b_{i,k_i}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})$$

$$=(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})-(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})-(\sum_{b_{i,k_i}<b_{i,j_i}\leqslant\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})$$

$$=(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}})-(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})$$

$$=(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}}}-\Omega_{e_i}^{b_{i,k_i}})$$

即$\hat{R}-\hat{R}'=(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}}-\Omega_{e_i}^{b_{i,k_i}})$得证；

由式(10)减式(6)，得

$$\check{R}'-\check{R}=\sum_{\bar{j}_i=1}^{\bar{k}_i-2}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))+(\sum_{\bar{b}_{i,\bar{k}_i-1}\leqslant b_{i,j_i}<b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})+\sum_{\bar{j}_i=\bar{k}_i}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))-\sum_{\bar{j}_i=1}^{\bar{n}_i}(\bar{p}_{i,\bar{j}_i}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{j}_i}}))$$

$$=(\sum_{\bar{b}_{i,\bar{k}_i-1}\leqslant b_{i,j_i}<b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})-\bar{p}_{i,\bar{k}_i-1}\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})$$

$$=(\sum_{\bar{b}_{i,\bar{k}_i-1}\leqslant b_{i,j_i}<b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})+(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})-(\sum_{\bar{b}_{i,\bar{k}_i-1}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})$$

$$=(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}})-(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})$$

$$=(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})(\Pr(\Omega_{e_i}^{b_{i,k_i}}-\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}}))$$

即$\check{R}'-\check{R}=(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}}-\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}})$得证。

引理3表明，每增加枚举边e_i一个中间状态无需重新枚举出所有网络有效状态，只需通过集合之间的比较运算，即可快速计算可靠度上下界的改变值。记可靠度上下界之差$dR=\hat{R}-\check{R}$，基于引理3可知，增加枚举边e_i的中间状态b_{i,k_i}后可靠度上下界之差的减少值ΔdR为：

$$\Delta dR=dR-dR'=(\hat{R}-\check{R})-(\hat{R}'-\check{R}')=(\hat{R}-\hat{R}')+(\check{R}'-\check{R})=(\sum_{\bar{b}_{i,\bar{k}_i-1}<b_{i,j_i}\leqslant b_{i,k_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i}}-\Omega_{e_i}^{b_{i,k_i}})+(\sum_{b_{i,k_i}\leqslant b_{i,j_i}<\bar{b}_{i,\bar{k}_i}}p_{i,j_i})\Pr(\Omega_{e_i}^{b_{i,k_i}}-\Omega_{e_i}^{\bar{b}_{i,\bar{k}_i-1}}) \tag{12}$$

式(12)表明，边e_i的中间状态b_{i,k_i}对可靠度的影响除取决于边处于该状态的概率外，还取决于该状态与相邻状态的转换对网络性能的影响。为枚举最少的状态得到最紧凑的上下界，应优先枚举边e_i中对应ΔdR最大的中间状态。

基于引理1—引理3，可通过不断增加枚举边的中间状态，来获得逐步收敛的可靠度上下界，思路为：(1)每条边仅取完全工作状态b_{i,n_i}和失效状态$b_{i,1}$，枚举对应的网络有效状态，分别将边处于中间状态b_{i,j_i}($1<j_i<n_i$)的概率p_{i,j_i}叠加到p_{i,n_i}和$p_{i,1}$上，计算网络有效状态概率之和，得到可靠度上下界的初始值；(2)分别计算增加枚举各边中未枚举的中间状

态后可靠度上下界之差的减小值 ΔdR，取最大 ΔdR 值对应的边的中间状态为下一步待枚举状态；(3)增加枚举上一步得到的待枚举状态，将随之增加的网络有效状态并入网络有效状态空间；(4)转向步骤(2)，直至可靠度上下界之差达到可接受数值。

4 算法实现及步骤

4.1 定义变量

对应边 $e_i(i=1,2,\cdots,|E|)$ 定义集合 $S_i=\{s_{i,0},s_{i,1},s_{i,2},\cdots,s_{i,j_i}\cdots,s_{i,n_i},s_{i,n_i+1}\}$，其中 $s_{i,j_i}\in\{0,1\}$；对于 $1\leqslant j_i\leqslant n_i$，规定 $s_{i,j_i}=0$ 表示边 e_i 的容量取值 b_{i,j_i} 未被枚举，$s_{i,j_i}=1$ 表示边 e_i 的容量取值 b_{i,j_i} 已被枚举，且 $s_{i,0}=s_{i,n_i+1}=1$。对应于某一容量取值 b_{i,j_i}，定义 $L(b_{i,j_i\neq 1})$ 和 $U(b_{i,j_i\neq n_i})$ 分别表示边 e_i 所有被枚举的容量取值中小于 b_{i,j_i} 的最大值和大于 b_{i,j_i} 的最小值，即 $L(b_{i,j_i\neq 1})=\max\limits_{b_{i,k_i}\in B_i,s_{i,k_i}=1,b_{i,k_i}<b_{i,j_i}}(b_{i,k_i})$，$U(b_{i,j_i\neq n_i})=\min\limits_{b_{i,k_i}\in B_i,s_{i,k_i}=1,b_{i,k_i}>b_{i,j_i}}(b_{i,k_i})$。

对应边 $e_i(i=1,2,\cdots,|E|)$ 定义 $\hat{P}_i=\{\hat{p}_{i,1},\hat{p}_{i,2},\cdots,\hat{p}_{i,j_i},\cdots,\hat{p}_{i,n_i}\}$ 和 $\check{P}_i=\{\check{p}_{i,1},\check{p}_{i,2},\cdots,\check{p}_{i,j_i}\cdots,\check{p}_{i,n_i}\}$ 分别表示计算可靠度上下界时边 e_i 对应的取值概率集合，其中

$$\hat{p}_{i,j_i}=\begin{cases}0, & s_{i,j_i}=0\\ p_{i,1}, & s_{i,j_i}=1,j_i=1\\ \sum\limits_{L(b_{i,j_i})<b_{i,k_i}\leqslant b_{i,j_i}}p_{i,k_i}, & s_{i,j_i}=1,1<j_i\leqslant n_i\end{cases}$$

$$\check{p}_{i,j}=\begin{cases}0, & s_{i,j_i}=0\\ \sum\limits_{b_{i,j_i}\leqslant b_{i,k_i}<U(b_{i,j_i})}p_{i,k_i}, & s_{i,j_i}=1,1\leqslant j_i<n_i\\ p_{i,n_i}, & s_{i,j_i}=1,j_i=n_i\end{cases}$$

对应边 $e_i(i=1,2,\cdots,|E|)$，定义 $3n_i-4$ 个 $|E|-1$ 维向量的集合 $\Omega_{e_i}^{b_{i,1}},\Omega_{e_i}^{b_{i,2}},\cdots,\Omega_{e_i}^{b_{i,j_i}},\cdots,\Omega_{e_i}^{b_{i,n_i}}$；$\hat{\Psi}_{i,2},\cdots,\hat{\Psi}_{i,j_i},\cdots,\hat{\Psi}_{i,n_i-1}$；$\check{\Psi}_{i,2},\cdots,\check{\Psi}_{i,j_i},\cdots,\check{\Psi}_{i,n_i-1}$；定义 $|E|$ 维向量的集合 Ω，$|E|-1$ 维向量的集合 $\Phi=\{Y_1,Y_2,\cdots,Y_l,\cdots,Y_{|\Phi|}\}$。

$\hat{R}$ 和 $\check{R}$ 分别表示网络可靠度的上下边界值；dR 表示可靠度上下界之差，即 $dR=\hat{R}-\check{R}$；$\Delta\hat{R}$、$\Delta\check{R}$ 和 ΔdR 分别表示每经过1步运算 $\hat{R}$ 的减少值、$\check{R}$ 的增加值以及 dR 的减少值；$0\leqslant\varepsilon_0\leqslant 1$ 表示可以接受的可靠度上下界之差，即当 $dR\leqslant\varepsilon_0$ 时，终止计算。

4.2 算法步骤

步骤0 初始化变量。

对应于 $i=1,2,\cdots,|E|$，令 $s_{i,1}=s_{i,n_i}=1$，$s_{i,2}=s_{i,3}=\cdots=s_{i,n_i-1}=0$，由 $U(b_{i,j_i})$、$L(b_{i,j_i})$、$\hat{P}_i$ 及 $\check{P}_i$ 定义可知 $L(b_{i,2})=L(b_{i,3})=\cdots=L(b_{i,n_i})=b_{i,1}$，$U(b_{i,1})=U(b_{i,2})=\cdots=U(b_{i,n_i-1})=b_{i,n_i}$，$\hat{P}_i=\{p_{i,1},0,\cdots,0,1-p_{i,1}\}$，$\check{P}_i=\{1-p_{i,n_i},0,\cdots,0,p_{i,n_i}\}$；令 $\Omega_{e_i}^{b_{i,1}}=\Omega_{e_i}^{b_{i,2}}=\cdots=\Omega_{e_i}^{b_{i,j_i}}=\cdots=\Omega_{e_i}^{b_{i,n_i}}=\hat{\Psi}_{i,2}=\cdots=\hat{\Psi}_{i,j_i}=\cdots=\hat{\Psi}_{i,n_i-1}=\check{\Psi}_{i,2},\cdots=\check{\Psi}_{i,j}=\cdots=\check{\Psi}_{i,n_i-1}=\Phi=\phi$；考查网络状态集合 $\{\boldsymbol{X}=(x_1,x_2,\cdots,x_i,\cdots,x_{|E|})\mid x_i\in\{b_{i,1},b_{i,n_i}\}\}$，生成有效网络状态集合 $\Omega=\{\boldsymbol{X}=(x_1,x_2,\cdots,x_i,\cdots,x_{|E|})\mid f(\boldsymbol{X})\geqslant d,x_i\in\{b_{i,1},b_{i,n_i}\}\}$。分别代入 $\hat{P}_i$、$\check{P}_i$，计算 $\Pr(\Omega)$，得可靠度上界 $\hat{R}$ 和可靠度下界 $\check{R}$，计算可靠度上下界之差 $dR=\hat{R}-\check{R}$；若 $dR\leqslant\varepsilon_0$，结束，否则，转向步骤1。

步骤1 更新集合 $\Omega_{e_i}^{b_{i,j_i}}$ $(i\in\{1,2,\cdots,|E|\},j_i\in\{1,2,\cdots,n_i\},s_{i,j_i}=1$ 且 $s_{i,j_i-1}s_{i,j_i+1}=0)$。

(1-1) $i=1$。

(1-2) $j_i=1$。

(1-3)若 $s_{i,j_i}=1$ 且 $s_{i,j_i-1}s_{i,j_i+1}=0$，则更新 $\Omega_{e_i}^{b_{i,j_i}}=\Omega/(e_i,b_{i,j_i})$，转向步骤(1-4)；否则，直接转向步骤(1-4)。

(1-4) $j_i=j_i+1$，若 $j_i\leqslant n_i$，则转向步骤(1-3)；否则，$i=i+1$，若 $i\leqslant|E|$，则转向步骤(1-2)，否则转向步骤2。

步骤2 更新集合 $\Omega_{e_i}^{b_{i,j_i}}$、$\hat{\Psi}_{i,j_i}$ 和 $\check{\Psi}_{i,j_i}$ $(i\in\{1,2,\cdots,|E|\},j_i\in\{1,2,\cdots,n_i\},s_{i,j_i}=0)$。

(2-1) $i=1$。

(2-2) $j_i=1$。

(2-3)若 $s_{i,j_i}=1$ 且 $s_{i,j_i+1}=0$，则对于 $b_{i,j_i}<b_{i,s_i}<U(b_{i,j_i})$：更新 $\Omega_{e_i}^{b_{i,s_i}}=\Omega_{e_i}^{b_{i,s_i}}\cup\Omega_{e_i}^{b_{i,j_i}}$；令 $\Phi=\Omega_{e_i}^{U(b_{i,j_i})}-\Omega_{e_i}^{b_{i,j_i}}$，若 $\Phi=\phi$，则转向步骤(2-8)，若 $\Phi\neq\phi$，则转向步骤(2-4)。否则，转向步骤(2-8)。

(2-4) $k_i=j_i+1$，转向步骤(2-5)。

(2-5)从 Φ 中任取一 $|E|-1$ 维向量 Y_l，计算 $f(b_{i,k_i}\cdot Y_l)$，如 $f(b_{i,k_i}\cdot Y_l)\geqslant d$，则对于 $b_{i,k_i}\leqslant b_{i,s_i}<U(b_{i,j_i})$：更新 $\Omega_{e_i}^{b_{i,s_i}}=\Omega_{e_i}^{b_{i,s_i}}\cup\{Y_l\}$，$\check{\Psi}_{i,s_i}=\check{\Psi}_{i,s_i}\cup\{Y_l\}$，转向步骤(2-7)；若 $f(b_{i,k_i}\cdot Y_l)<d$，更新 $\hat{\Psi}_{i,s_i}=\hat{\Psi}_{i,s_i}\cup\{Y_l\}$，则转向步骤(2-6)；

(2-6) $k_i=k_i+1$，若 $b_{i,k_i}<U(b_{i,j_i})$，则转向步骤(2-5)；否则，转向步骤(2-7)。

(2-7)从 Φ 中删除 Y_l，即令 $\Phi=\Phi-\{Y_l\}$。若 $\Phi\neq\phi$，则转向步骤(2－5)；否则，转向步骤(2-8)。

(2-8) $j_i=j_i+1$，若 $j_i<n_i$，则转向步骤(2-3)；否则，转向步骤(2-9)。

(2-9) $i=i+1$，若 $i\leqslant|E|$，则转向步骤(2-2)；否则，转向步骤3。

步骤3 计算 $\Pr(\hat{\Psi}_{i,j_i})$ 和 $\Pr(\check{\Psi}_{i,j_i})$ $(i\in\{1,2,\cdots,|E|\},j_i\in\{1,2,\cdots,n_i\},s_{i,j_i}=0)$，更新 $\hat{R}$、$\check{R}$ 和 dR。

(3-1) $\Delta\hat{R}=\Delta\check{R}=\Delta dR=0$。

(3-2) $i=1$。

(3-3) $j_i=2$。

(3-4)若 $s_{i,j_i}=0$，计算 $(\sum\limits_{L(b_{i,j_i})<b_{i,k_i}\leqslant b_{i,j_i}}p_{i,k_i})\Pr(\hat{\Psi}_{i,j_i})$ 和 $(\sum\limits_{b_{i,j_i}\leqslant b_{i,k_i}<U(b_{i,j_i})}p_{i,k_i})\Pr(\check{\Psi}_{i,j_i})$，若 $(\sum\limits_{L(b_{i,j_i})<b_{i,k_i}\leqslant b_{i,j_i}}p_{i,k_i})\Pr(\hat{\Psi}_{i,j_i})+(\sum\limits_{b_{i,j_i}\leqslant b_{i,k_i}<U(b_{i,j_i})}p_{i,k_i})\Pr(\check{\Psi}_{i,j_i})>\Delta dR$，则 $\Delta\hat{R}=(\sum\limits_{L(b_{i,j_i})<b_{i,k_i}\leqslant b_{i,j_i}}p_{i,k_i})\Pr(\hat{\Psi}_{i,j_i})$，$\Delta\check{R}=(\sum\limits_{b_{i,j_i}\leqslant b_{i,k_i}<U(b_{i,j_i})}p_{i,k_i})\Pr$

$(\check{\Psi}_{i,j_i})$，$\Delta dR=\Delta\hat{R}+\Delta\check{R}$，同时将 b_{i,j_i} 选为下一步枚举对象；若 $s_{i,j_i}=1$，则转向步骤(3-5)。

(3-5) $j_i=j_i+1$，若 $j_i<n_i$，则转向步骤(3-4)；否则，转向步骤(3-6)。

(3-6) $i=i+1$，若 $i\leqslant|E|$，则转向步骤(3-3)；否则，转向步骤(3-7)。

(3-7) $\hat{R}=\hat{R}-\Delta\hat{R}$，$\check{R}=\check{R}+\Delta\check{R}$，$dR=dR-\Delta dR$。若 $dR\leqslant\varepsilon_0$，则终止计算；否则，转向步骤 4。

步骤 4　根据选定的枚举对象 b_{i,j_i} 更新 $\hat{\Psi}_{i,k_i}(L(b_{i,j_i})<b_{i,k_i}<b_{i,j_i})$、$\check{\Psi}_{i,k_i}(b_{i,j_i}<b_{i,k_i}<U(b_{i,j_i}))$；$\hat{p}_{i,j_i}$、$\check{p}_{i,j_i}$、$\hat{p}_{i,k_i}(b_{i,k_i}=U(b_{i,j_i}))$、$\check{p}_{i,k_i}(b_{i,k_i}=L(b_{i,j_i}))$；$s_{i,j_i}$、$L(b_{i,k_i})(b_{i,j_i}<b_{i,k_i}\leqslant U(b_{i,j_i}))$、$U(b_{i,k_i})(L(b_{i,j_i})\leqslant b_{i,k_i}<b_{i,j_i})$ 和 Ω。

(4-1)对于 $L(b_{i,j_i})<b_{i,k_i}<b_{i,j_i}$：$\hat{\Psi}_{i,k_i}=\hat{\Psi}_{i,k_i}-\hat{\Psi}_{i,j_i}$；对于 $b_{i,j_i}<b_{i,k_i}<U(b_{i,j_i})$：$\check{\Psi}_{i,k_i}\check{\Psi}_{i,k_i}-\check{\Psi}_{i,j_i}$。

(4-2) $\hat{p}_{i,j_i}=\sum_{L(b_{i,j_i})<b_{i,k_i}\leqslant b_{i,j_i}} p_{i,k_i}$，$\check{p}_{i,j_i}=\sum_{b_{i,j_i}\leqslant b_{i,k_i}<U(b_{i,j_i})} p_{i,k_i}$；对于 $b_{i,k_i}=U(b_{i,j_i})$：$\hat{p}_{i,k_i}=\hat{p}_{i,k_i}-\hat{p}_{i,j_i}$；对于 $b_{i,k_i}=L(b_{i,j_i})$：$\check{p}_{i,k_i}=\check{p}_{i,k_i}-\check{p}_{i,j_i}$。

(4-3) $s_{i,j_i}=1$；对于 $L(b_{i,j_i})\leqslant b_{i,k_i}<b_{i,j_i}$：$U(b_{i,k_i})=b_{i,j_i}$；对于 $b_{i,j_i}<b_{i,k_i}\leqslant U(b_{i,j_i})$：$L(b_{i,k_i})=b_{i,j_i}$。

(4-4) $\Omega=b_{i,j_i}\cdot\Omega_{e_i}^{b_{i,j_i}}$。

(4-5)转向步骤 1。

5　算例分析

多状态网络 $G(V,E)$ 如图 1 所示[6,7,15,16,22]。网络中节点集合 $V=\{v_1(s),v_2,v_3,v_4(t)\}$，边集合 $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$；各边的容量取值集合分别为 $B_1=\{0,1,2\}$，$B_2=\{0,1,2,3\}$，$B_3=\{0,1,2\}$，$B_4=\{0,1,2,3\}$，$B_5=\{0,1,2,3\}$，$B_6=\{0,1,2,3\}$，对应的取值概率如表 1 所列；源点 s 和汇点 t 之间的需求流量 $d=4$。

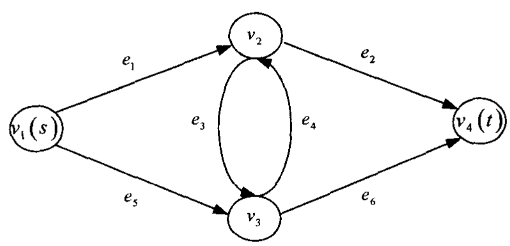

图 1　一个含 4 个节点、6 条边的多状态网络

表 1　各边的取值概率[15,16,22]

p_i	$p_{i,1}$	$p_{i,2}$	$p_{i,3}$	$p_{i,4}$
p_1	0.10	0.30	0.60	-
p_2	0.10	0.30	0.30	0.30
p_3	0.10	0.50	0.40	-
p_4	0.05	0.25	0.40	0.30
p_5	0.10	0.30	0.20	0.40
p_6	0.10	0.20	0.45	0.25

为便于算法比较，令 $\varepsilon_0=0$。利用本文提出的算法按照不同迭代步数得出的可靠度上下界及上下界之差如图 2 所示。从图 2 可看出，可靠度的上下界随迭代计算步数的增加逐步收敛于精确值，对应上下界之差逐步收敛于 0。因此，算法执行到一定程度后，当上下界之差足够小时，即可用可靠度上下界的平均值来近似估算其真实值。

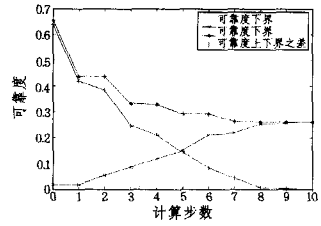

图 2　本文算法得到的可靠度上、下界及上下界之差

利用空间截尾法及本文算法得到的可靠度上下界及上下界之差与需枚举的网络状态数量的关系分别如图 3、图 4 所示。从图中可看出，枚举同样多的网络状态，本文算法可得到更为紧凑的上下界，且收敛于精确值所需枚举的网络状态的数量(155 个)仅占空间截尾法(2304 个)的 6.7%，其远远小于空间截尾法。

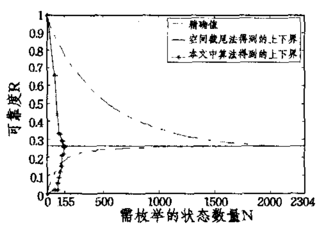

图 3　可靠度上下界与需枚举的网络状态数量的关系

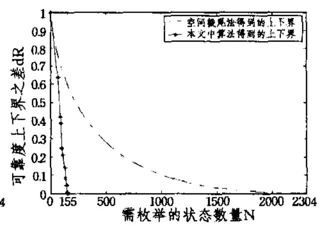

图 4　可靠度上下界之差与需枚举的网络状态数量的关系

结束语　本文提出了一种基于边状态枚举计算多状态网络可靠度动态界的算法。其主要特点有：(1)算法不需要提前求取网络 d-最小割(路)集；(2)随着迭代计算的深入，算法同时得到逐步减小的可靠度上界和逐步增加的可靠度下界，且最终均收敛于可靠度精确值；(3)枚举较少的网络状态即可得到较为紧凑的可靠度上下界，具有较高的计算效率。

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